Since in forming the pair of elements one and only one element of a pair can be taken from the same set of planes, it follows that $n(n-1)$ pairs can be taken from a same two sets of planes in which a corresponding pair of planes in the set are kept fixed and, therefore, in every possible way there can be formed

$$n(n-1)n_2 \text{ or } \frac{n^2\ (n-1)^2}{2\,!}$$

by choosing in every possible manner the pair of planes to be kept fixed.

It should be noticed that the expansion passes directly from those terms involving only $n-2$ elements of the principal diagonal plane to those involving them all. There would be no difficulty in stating at once the general law for the expansion of a cubical determinant in terms of the elements of the principal diagonal of the principal diagonal plane of the cubical determinant and their coaxial cubical minors.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

256. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Three men, A, B, and C, rented a pasture for a fixed amount, each to pay per month in proportion to the stock pastured. During the first month A put in 3 horses and B and C each some horses, and B paid for the month \$6, but A and C each defaulted payment. During the next month each put in one more horse, and C paid for the month \$7.20, but A and B each defaulted payment. During the next month each put in one more horse, and A paid his bill for the month, \$5, but B and C each defaulted.

Required: (1) the rent of the pasture per month; (2) the number of horses B and C each put in during the first month; and (3) the amount A, B, and C, each, owed for the unpaid service.

I. Solution by S. A. COREY, Hiteman, Iowa.

Let a, b, c=rate per horse per month for the first, second, and third month, respectively. Let x, y=number of horses put in the first month by B, and C, respectively. Let $n=3+x+y$=total number of horses in pasture first month. Let m=fixed monthly rental of pasture.

Then as A paid \$5 for the third month's rental when he had 5 horses in the pasture, c=\$1. As fewer horses were in the pasture the two preceding months the rate per horse per month was more than \$1 for the first and second months,

or both a and $b>\$1$. Then as x is an integer, as $a>\$1$, and as B paid but \$6 for the first month's service, $x \gtreqless 5$. Similarly, as y is an integer, $b>\$1$, and as C paid but \$7.20 for the second month's service $(y+1) \gtreqless 7$, or $y \gtreqless 6$. Hence as $n=3+x+y$, $n \gtreqless 14$. But as $c=m/(n+6)=1$, $m=(n+6)$, whence $m \gtreqless \$20$. But as not less than 5 horses were placed in the pasture the first month, $m= \gtreqless 11$.

B had x horses in the pasture the first month, the rate then being m/n, or $m/(m-6)$, and paid \$6 for the service. Hence, $xm/(m-6)=36$, or reducing and transposing, $(6-x)m=36$, but as $(6-x)$=integer, and $11 \gtreqless m \gtreqless 20$ (remembering that C had some horses in the pasture the first month, and that $n=m-6$). $6-x=2$, $m=\$18$, whence $x=4$ and $n=12$. But $y=n-3-x=5$. As $a=m/n=$ \$1.50, $b=m/(n+3)=\$1.20$, and $c=\$1$, we readily find that

A owed \$4.50 for the first, and \$4.80 for the second month's service=\$9.30 total.
B owed \$6 for the second, and \$6 for the third month's service=\$12 total.
C owed \$7.50 for the first, and \$7 for the third month's service=\$14.50 total.

II. Solution by REV. J. H. MEYER, S. J., Professor of Mathematics, College of the Sacred Heart, Augusta, Ga.; by M. R. BECK, Cleveland, Ohio, and by J. EDWARD SANDERS, Reinersville, Ohio.

Using r as the number of dollars in one month's rent; x and y as the number of horses B and C put in at first, respectively, we obtain

$$\frac{rx}{3+x+y}=6, \text{ or } r=\frac{18+6x+6y}{x}. \tag{1}$$

$$\frac{r(y+1)}{6+x+y}=7.20, \text{ or } r=\frac{43.2+7.2x+7.2y}{y+1}. \tag{2}$$

$$\frac{5r}{9+x+y}=5, \text{ or } r=\frac{45+5x+5y}{5}. \tag{3}$$

Equating (1) and (3), $90-15x+30y=5x^2+5xy$.........(4). Equating (2) and (3), $171+31x-14y=5xy+5y^2$..........(5). Adding (4) and (5), $-5(x+y)^2+16(x+y)=-261$..........(6). Solving (6) for $x+y$, $x+y=9$..........(7). Substituting (7) in (3), (2), (1), $r=18$, $y=5$, $x=4$. For first month, \$1.50, for the second, \$1.20, and for the third, \$1.00 must be paid for each horse.

Hence, A owed \$9.30; B, \$12.00; C, \$14.50.

Also solved by O. L. Callecot, A. H. Holmes, L. E. Newcomb, J. Scheffer, G. B. M. Zerr, and the Proposer.

257. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

Solve (1) $x+y=10$, (2) $3x=\log_{10}y$.

I. Solution by HENRY HEATON, Belfield, N. D.

We have $3x=\log_{10}(10-x)$, or $10-x=10^{3x}$. If we suppose $x=\frac{1}{3}$, we obtain $9\frac{2}{3}=10$. An error of $\frac{1}{3}$. If we suppose $x=\frac{1}{4}$, we get $9.25=4.38$. An error of 4.87. By position we obtain $x=.328$. Substituting this we get $9.672=9.639$. An error of $-.043$. Substituting .329 for x we get $9.671=9.704$. An error of